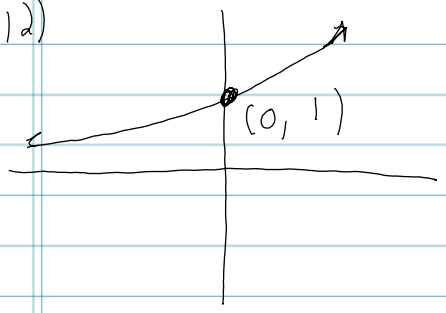
February 22 — Prof. Kincade

Section 3.3 & 3.5

12)



a) Is this a function?
↳ Yes

b) Domain?
↳ All real numbers {Every 'x' has some value of 'y'}

c) Range?
↳ $(0, \infty)$ {we can never get a value ≤ 0, so the range is ...}

d) No symmetry about x or y

e) Only one intercept; (0, 1), which is a y-intercept.

★ How can we tell if an equation is a function?

→ $3x^2 + 4y^2 = 12$

↳ Use a library of functions to think about its picture.

↳ Think about your inputs and outputs; does one x lead to exactly one y??

$3x^2 + 4y^2 = 12$

$4y^2 = -3x^2 + 12$

$y^2 = -\frac{3}{4}x^2 + 3$

$y = \pm\sqrt{-\frac{3}{4}x^2 + 3}$ → Two answers! Thus, this is not a function!

Section 3.3 – Properties of a function

1) odd or even?
2) Increasing or decreasing?
3) mins and maxes
4) secant lines!

} This will help you understand what is going on to the right of that $f(x) = \ldots$!

→ Odds and Evens

- General Test

1) Evaluate $f(-x)$

→ $f(-x) = -f(x)$ → odd function [origin symmetry]

→ $f(-x) = f(x)$ → Even function [y-axis symmetry]

→ $f(-x)$ = Some different function → Neither

★ $f(x) = x^2 + 3$ { Is $f(x)$ odd, even, or neither?

→ $f(-x) = (-x)^2 + 3 = x^2 + 3$

↳ $f(-x) = f(x)$ ⟷ Even function!

★ $f(x) = 2x^3 - 5x$

↳ $f(-x) = 2(-x)^3 - 5(-x)$

$= 2(-x^3) + 5x$

$= -2x^3 + 5x \Rightarrow -(2x^3 - 5x) = f(x)$

↳ $f(-x) = -f(x) \Rightarrow$ odd function!

★ $f(x) = x^2 + x - 1$

↳ $f(-x) = (-x^2) + (-x) - 1$

$= x^2 - x - 1$ ⟶ $-(-x^2 + x + 1)$

↳ It's not the same... ↳ And factoring a -1 out isn't the same either.

↳ $f(-x)$ = Something... but not what we're looking for. It is neither!

- $f(x) = \frac{x}{x^2-1}$

$f(-x) = \frac{(-x)}{((-x)^2)-1} = \frac{-x}{x^2-1} = \frac{-1}{1}\left(\frac{x}{x^2-1}\right)$

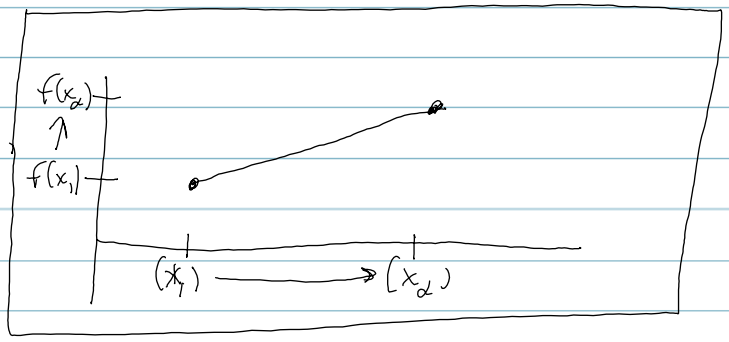
$f(-x) = -f(x) \Rightarrow$ The function is odd!

★ The key point is: $\frac{-1}{-1} = \frac{1}{1} = \frac{-1}{1}$

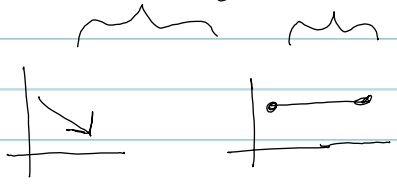
$-1 \cdot \frac{-1}{-1} = 1$; $-1/-1 = 1$!

Increasing / Decreasing

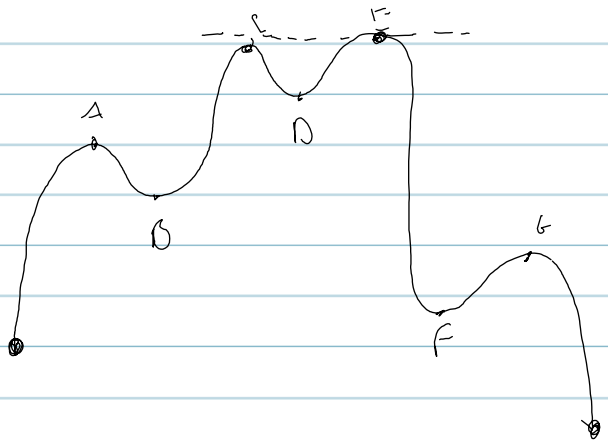
- A function is increasing on an open interval I, if for any choice of x_1 and x_2 in I, with $x_1 < x_2$, then $f(x_1) < f(x_2)$



- Try to come up with this definition on your own to describe decreasing / constant functions.



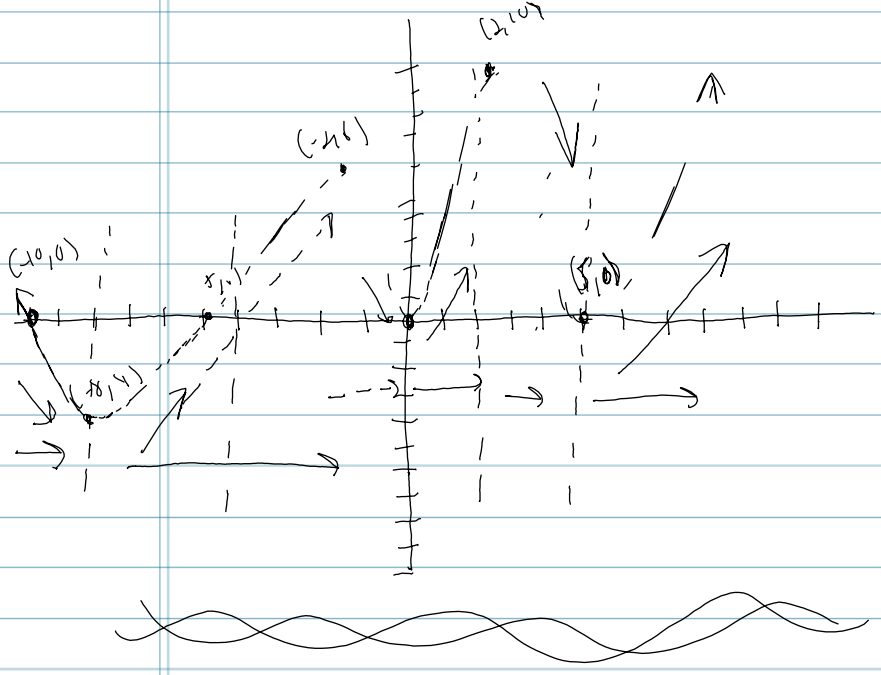
Local (relative) mins & maxes



"Valleys", or "Local points that are smallest"
→ B, D, F ⇒ mins, relative

"Peaks", or "highest points in a region"
→ A, C, E, G ⇒ maxes, relative

E ⇒ ABSOLUTE max; no other point has a greater value.



12) Is the function decreasing on the interval $(-8, -4)$ (x_1, x_2) → Interval, NOT a point!

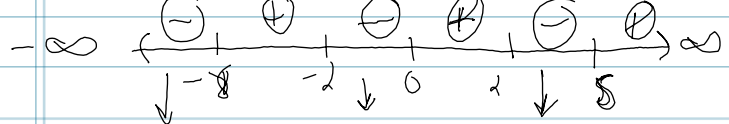
→ No! The function is INCREASING on this interval; the y-values are getting larger.

14) Is f decreasing on the interval $(2, 5)$

→ Yes! The y-values are getting smaller as x changes from 2 to 5.

16) List the intervals on which F is decreasing.

mins and maxes!



$(-\infty, -8) \cup (-2, 0) \cup (2, 5)$

(x = 5)

18) Is there a local maximum at 5? If so, what is it?

↳ The "5" is an x value, so...

↳ Does x=5 yield a max? NO

↳ However, it is a min, and the min is 0.

The point is (5, 0), but do not confuse the x for the min max, or vice-versa.

20) List where 'f' has a local minimum. [The x-values!]
What are the local minimums? [The y-values!]

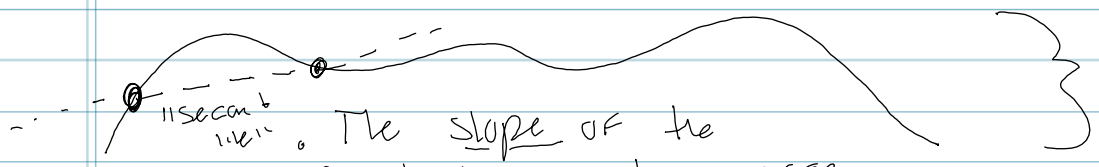
* minima / maxima = plural for values.

↳

x	y
−8	−11
0	0
3	0

→ −11 is the absolute minimum!

* secant lines



your speed

. The slope of the secant line is the average rate of change for that interval.

Section 3.5 – Transformations

- $y = x \Rightarrow$
- $y = x^3 \Rightarrow$
- $y = \sqrt{x} =$
- $y = x^n \Rightarrow$
- $y = |x| \Rightarrow$

• $y = -2|x-1|+4$ ★ base shape?

$a|x-h|+k$

(1, 4)

• $a = -2$
$-1 \Rightarrow$ down
$2 \Rightarrow$ stretched

★ Final shape?

(1, 4)

x	y
-1	0 → a
0	2 → b
1	4 → c
2	2 → d
3	0 → e

Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$

• $y = -(x-1)^3 + 2$ → Bas. Shape?

$a = \pm 1 \Rightarrow$
$h = 1$
$k = 2$

x	y
0	3 → A
1	2 → B
2	1 → C

Domain $\Rightarrow (-\infty, \infty)$
Range $\Rightarrow (-\infty, \infty)$

★ $y = a\sqrt{x-h} + k$ → $y = \sqrt{3-x} + 2$

left and right

$h = 3$, $k = 2$

$a = -1 \Rightarrow$ down & neutral
k:
$+x \Rightarrow$ right, $-x \Rightarrow$ left